## 5th Annual Lexington Math Tournament Theme Round

March 28, 2015 Name \_\_\_\_\_ Team \_\_\_\_\_ Music 1. \_\_\_\_\_ 2. 3. \_\_\_\_\_ 4. \_\_\_\_\_ 5. \_\_\_\_\_ Physics 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_ 4. \_\_\_\_\_ 5. \_\_\_\_\_ Frisbee 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_ 4. \_\_\_\_\_ 5. \_\_\_\_\_

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## 1 Songs

- 1. Meghan Trainor is all about those base-systems. If she weighs 451 pounds in some base, and 127 pounds in a base that is twice as big, how much does she weigh in base 10?
- 2. Taylor Swift made the song 15 in 2008, and the song 22 in 2012. If she had continued at the same rate, in what year would her song 1989 have come out?
- 3. Sir Mix-A-Lot likes big butts and cannot lie. He is with his three friends, who all always lie. If each of the following statements was said by a different person, who is Sir Mix-A-Lot?
  - A: B and D both like big butts.
  - B: C and D either both like big butts, or neither of them do.

C: B does not like big butts.

- D: A or C (or both) like big butts.
- 4. Mark Ronson is going to give some uptown funk-tions to you. Bruno Mars is at 1 degree Fahrenheit right now (that is, time, in seconds, t = 0), and his heat h(t) is given by  $h(t) = 2 \cdot h(t-1) + t$ . If he will explode when he is over 1,000,000 degrees Fahrenheit, at what time t will Bruno Mars explode?
- 5. Jay-Z, who is a emcee, took 4 AMC contests, for a total of 100 problems. He got 99 problems correct and 1 problem wrong. His answer, z, to the problem he missed, was the square root of the correct 4-digit answer, j. If z is also 3 times the sum of the digits of j, then what is the ordered tuple (j, z)?

## 2 Physics

- 1. Two buildings are connected by a rope. The rope is 20 meters long and each end is connected to the top corner of a building. If the rope droops 10 meters below the roofs of the buildings, then how far away are the buildings from each other?
- 2. A ball bounces up and down and bounces up 80% of its height after each bounce. If it is dropped from a height of 2 feet above the ground, what is the total distance the ball will travel before coming to rest on the ground?
- 3. The formula for the gravitational acceleration on a planet is  $\frac{Gm}{r^2}$  where G is a constant, m is the mass of the planet, and r is the radius of the planet. If a certain planet has half the radius of Earth, and a third of the mass, what is the ratio of Earth's gravitational constant to that planet's?
- 4. Albert knows that there are 6 types of leptons and 6 types of quarks, but he doesn't know the name of any of them. On a physics test, Albert is given a list of the names of all 12 particles, and has to label 6 as leptons and 6 as quarks. What is the probability that he gets at least 10 of the 12 particles correctly on the test?
- 5. 3 planets are orbiting around a sun in coplanar orbits. One planet makes 1 orbit every 21 years, one makes an orbit every 33 years, and one makes an orbit every 77 years. If all of the planets and the sun lie along the same line right now, how long, in years, will it be before all 4 lie along the same line again?

## 3 Frisbees

In the following problems, some number of kids are **evenly spaced** around the perimeter of a **unit circle**. One frisbee is being thrown from kid to kid, taking the path of the line in between them. To throw the frisbee "5 kids left", for example, would mean to throw it in a **straight line** from the current kid to one 5 spaces in the clockwise direction, skipping over 4 kids.

- 1. There are 2015 kids, and each time the frisbee is thrown, it is thrown 100 kids left. How many kids will be guaranteed never be able to touch the frisbee?
- 2. Call the probability that a pass made by a certain player is not caught, that player's "Likelihood of a Missed Throw", or LMT for short. If Ivan, who is standing in a circle with 9 other players, has an LMT of 90%, and each player's LMT is the average of the LMTs of the people on their left and right, then what is the likelihood that the frisbee will make it all the way back around to Ivan?
- 3. If Zach threw the frisbee to his friend Clive, who is standing 3 spaces to Zach's left, the frisbee would go half as far as if he threw it to Henry, who is standing 9 spaces to Zach's left. How far would the frisbee go if Clive were to throw it to Henry?
- 4. Throwing the frisbee a distance d takes a total time of  $d^2$  seconds. Moreover, after a person catches a frisbee, it takes them one second before they can throw it. Suppose Steven is in a circle with 12 total people, and starts by throwing the frisbee. It goes clockwise around the circle back to Steven in N throws, with him catching it after 11 seconds. What is the sum of all the possible values of N?
- 5. If there are 187 kids in the circle, and each kid will either pass the frisbee 17 spaces to their left, or 11 spaces to their right, then how many ways are there to pass the frisbee 187 times, such that every person has thrown the frisbee once and every person has caught the frisbee once?